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Rough Neutrosophic Multisets

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Abstract. Many past studies largely described the concept of neutrosophic sets, neutrosophic multisets, rough sets, and rough neutrosophic sets in many areas. However, no paper has discussed about rough neutrosophic multisets. In this paper, we present some definition of rough neutrosophic multisets such as complement, union and intersection. We also have examined some desired properties of rough neutrosophic multisets based on these definitions. We use the hybrid structure of rough set and neutrosophic multisets since these theories are powerful tool for managing uncertainty, indeterminate, incomplete and imprecise information.

Keywords: Neutrosophic set, neutrosophic multiset, rough set, rough neutrosophic set, rough neutrosophic multisets

1 Introduction

In our real-life problems, there are situations with uncertain data that may be not be successfully modelled by the classical mathematics. For example, the opinion about "beauty", which is can be describe by more beauty, beauty, beauty than, or less beauty. Therefore, there are some mathematical tools for dealing with uncertainties such as fuzzy set theory introduced by Zadeh [1], intuitionistic fuzzy set theory introduced by Attanasov [2], rough set theory introduced by Pawlak [3], and soft set theory initiated by Molodtsov [4]. Rough set theory introduced by Pawlak in 1981/1982, deals with the approximation of sets that are difficult to describe with the available information. It is expressed by a boundary region of set and also approach to vagueness. After Pawlak's work several researcher were studied on rough set theory with applications [5], [6].

However, these concepts cannot deal with indeterminacy and inconsistent information. In 1995, Smarandache [7] developed a new concept called neutrosophic set (NS) which generalizes probability set, fuzzy set and intuitionistic fuzzy set. There are three degrees of membership described by NS which is membership degree, indeterminacy degree and non-membership degree. This theory and their hybrid structures has proven useful in many different field [8], [9], [10], [11],[12], [13],[14].

Broumi et al. [15] proposed a hybrid structure called neutrosophic rough set which is combination of neutrosophic set [7] and rough set [3] and studied their properties. Later, Broumi et al. [16] introduced interval neutrosophic rough set that combines interval- valued neutrosophic sets and rough sets. It studies roughness in interval- valued neutrosophic sets and some of its properties. After the introduction of rough neutrosophic set theory, many interesting application have been studied such as in medical organisation [17], [18], [19].

But until now, there have been no study on rough neutrosophic multisets (RNM). Therefore, the objective of this paper is to study the concept of RNM which is combination of rough set [3] and neutrosophic multisets [20] as a generalization of rough neutrosophic sets [15].

This paper is arranged in following manner. In section 2, some mathematical preliminary concepts were recall for more understanding about RNM. In section 3, the concepts of RNM and some of their properties are presented with examples. Finally, we conclude the paper.

2 Mathematical Preliminaries

In this section, we mainly recall some notions related to neutrosophic sets [7], [21], [22], neutrosophic multisets [23], [24], [20], [25], rough set [3], and rough neutrosophic set [15], [17], that relevant to the present work and for further details and background.

Definition 2.1 (Neutrosophic Set) [7] Let X be an universe of discourse, with a generic element in X denoted by x, the neutrosophic (NS) set is an object having the form

$$A = \{\langle x, (T_A(x), I_A(x), F_A(x))\rangle \mid x \in X\}$$

where the functions $T, I, F : X \to]^-0, 1^+[$ define respectively the degree of membership (or Truth), the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element $x \in X$ to the set A with the condition

$$^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$$

From a philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. So, instead of $]^-0, 1^+[$ we need to take the interval [0, 1] for technical applications, because $]^-0, 1^+[$ will be difficult to apply in the real applications such as in scientific and engineering problems. Therefore, we have

$$A = \{\langle x, (T_A(x), I_A(x), F_A(x))\rangle \mid x \in X,\ T_A(x), I_A(x), F_A(x) \in [0, 1]\}.$$

There is no restriction on the sum of $T_A(x)$; $I_A(x)$ and $F_A(x)$, so

$$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$$

For two NS,

$$A = \{\langle x, (T_A(x), I_A(x), F_A(x))\rangle \mid x \in X\} \text{ and}$$

$$B = \{\langle x, (T_B(x), I_B(x), F_B(x))\rangle \mid x \in X\}$$

the relations are defined as follows:

(i) $A \subseteq B$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$,

(ii) $A = B$ if and only if $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$,

(iii) $A \cap B = \{\langle x, \min(T_A(x),T_B(x)), \max(I_A(x),I_B(x)), \max(F_A(x),F_B(x))\rangle \mid x \in X\}$,

(iv) $A \cup B = \{\langle x, \max(T_A(x),T_B(x)), \min(I_A(x),I_B(x)), \min(F_A(x),F_B(x))\rangle \mid x \in X\}$,

(v) $A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle \mid x \in X\}$

(vi) $0_n = (0, 1, 1)$ and $1_n = (1, 0, 0)$.

As an illustration, let us consider the following example.

Example 2.2. Assume that the universe of discourse $U = \{x_1, x_2, x_3\}$, where x_1 characterizes the capability, x_2 characterizes the trustworthiness and x_3 indicates the prices of the objects. It may be further assumed that the values of x_1, x_2, and x_3 are in [0, 1] and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components which is the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is a neutrosophic set (NS) of U, such that,

$$A = \{\langle x_1, (0.3, 0.4, 0.5)\rangle, \langle x_2, (0.5, 0.1, 0.4)\rangle, \langle x_3, (0.4, 0.3, 0.5)\rangle\},$$

where the degree of goodness of prices is 0.4, degree of indeterminacy of prices is 0.3 and degree of poorness of prices is 0.5 etc.

The following definitions are refer to [25].

Definition 2.3 (Neutrosophic Multisets) Let E be a universe. A neutrosophic multiset (NMS) A on E can be defined as follows:

$$A = \{\langle x, (T_A^1(x), T_A^2(x), \ldots, T_A^p(x)), (I_A^1(x), I_A^2(x), \ldots, I_A^p(x)), (F_A^1(x), F_A^2(x), \ldots, F_A^p(x))\rangle : x \in E\}$$

where, the truth membership sequence $(T_A^1(x), T_A^2(x), \ldots, T_A^p(x))$, the indeterminacy membership sequence $(I_A^1(x), I_A^2(x), \ldots, I_A^p(x))$ and the falsity membership sequence $(F_A^1(x), F_A^2(x), \ldots, F_A^p(x))$ may be in decreasing or increasing order, and the sum of $T_A^i(x), I_A^i(x), F_A^i(x) \in [0,1]$ satisfies the condition $0 \le T_A^i(x) + I_A^i(x) + F_A^i(x) \le 3$ for any $x \in E$ and $i = 1, 2, \ldots, p$. Also, p is called the dimension (cardinality) of NMS A.

For convenience, a NMS A can be denoted by the simplified form:

$$A = \{\langle x, (T_A^i(x), I_A^i(x), F_A^i(x))\rangle \mid x \in E,\ i = 1,2,\ldots,p\}$$

Definition 2.4 Let $A, B \in \text{NMS}(E)$. Then,

(i) A is said to be NM subset of B is denoted by $A \tilde{\subseteq} B$ if $T_A^i(x) \le T_B^i(x)$, $I_A^i(x) \ge I_B^i(x)$, $F_A^i(x) \ge F_B^i(x)$, $\forall x \in E$.

(ii) A is said to be neutrosophic equal of B is denoted by $A = B$ if

$$T_A^i(x) = T_B^i(x),\ I_A^i(x) = I_B^i(x),\ F_A^i(x) = F_B^i(x),\ \forall x \in E.$$

(iii) The complement of A denoted by $A^{\tilde{C}}$ is defined by

$$A^{\tilde{C}} = \{\langle x, (F_A^1(x), F_A^2(x), \ldots, F_A^p(x)), (1 - I_A^1(x), 1 - I_A^2(x), \ldots, 1 - I_A^p(x)), (T_A^1(x), T_A^2(x), \ldots, T_A^p(x))\rangle : x \in E\}$$

(iv) If $T_A^i(x)=0$ and $I_A^i(x)=F_A^i(x)=1$ for all $x\in E$ and $i=1,2,\ldots,p,$ then A is called null ns-set and denoted by $\widetilde{\Phi}$.

(iv) If $T_A^i(x)=1$ and $I_A^i(x)=F_A^i(x)=0$ for all $x\in E$ and $i=1,2,\ldots,p,$ then A is called universal ns-set and denoted by $\widetilde{E}$.

Definition 2.5 Let $A,B\in$ NMS(E). Then,

(i) The union of A and B is denoted by $A\widetilde{\bigcup}B=C$ is defined by

$$C=\{\langle x,(T_C^1(x),T_C^2(x),\ldots,T_C^p(x)),\\ (I_C^1(x),I_C^2(x),\ldots,I_C^p(x)),\\ (F_C^1(x),F_C^2(x),\ldots,F_C^p(x))\rangle : x\in E\}$$

where

$$T_C^i(x)=T_A^i(x)\vee T_B^i(x),\quad I_C^i(x)=I_A^i(x)\wedge I_B^i(x),\\ F_C^i(x)=F_A^i(x)\wedge F_B^i(x),$$

for $\forall x\in E$ and $i=1,2,\ldots,p$.

(ii) The intersection of A and B is denoted by $A\widetilde{\bigcap}B=D$ and is defined by

$$D=\{\langle x,(T_D^1(x),T_D^2(x),\ldots,T_D^p(x)),\\ (I_D^1(x),I_D^2(x),\ldots,I_D^p(x)),\\ (F_D^1(x),F_D^2(x),\ldots,F_D^p(x))\rangle \mid x\in E\}$$

where

$$T_D^i(x)=T_A^i(x)\wedge T_B^i(x),\quad I_D^i(x)=I_A^i(x)\vee I_B^i(x),\\ F_D^i(x)=F_A^i(x)\vee F_B^i(x),$$

for $\forall x\in E$ and $i=1,2,\ldots,p$.

(iii) The addition of A and B is denoted by $A\widetilde{+}B=G$ and is defined by

$$G=\{\langle x,(T_G^1(x),T_G^2(x),\ldots,T_G^p(x)),\\ (I_G^1(x),I_G^2(x),\ldots,I_G^p(x)),\\ (F_G^1(x),F_G^2(x),\ldots,F_G^p(x))\rangle \mid x\in E\}$$

where

$$T_G^i(x)=T_A^i(x)+T_B^i(x)-T_A^i(x)\cdot T_B^i(x),\\ I_G^i(x)=I_A^i(x)\cdot I_B^i(x),\\ F_G^i(x)=F_A^i(x)\cdot F_B^i(x),$$

for $\forall x\in E$ and $i=1,2,\ldots,p$.

(iv) The multiplication of A and B is denoted by $A\widetilde{\times}B=H$ and is defined by

$$H=\{\langle x,(T_H^1(x),T_H^2(x),\ldots,T_H^p(x)),\\ (I_H^1(x),I_H^2(x),\ldots,I_H^p(x)),\\ (F_H^1(x),F_H^2(x),\ldots,F_H^p(x))\rangle \mid x\in E\}$$

where

$$T_H^i(x)=T_A^i(x)\cdot T_B^i(x),\\ I_H^i(x)=I_A^i(x)+I_B^i(x)-I_A^i(x)\cdot I_B^i(x),\\ F_H^i(x)=F_A^i(x)+F_B^i(x)-F_A^i(x)\cdot F_B^i(x),$$

for $\forall x\in E$ and $i=1,2,\ldots,p$.

Here $\wedge,\vee,+,\cdot,-$ denotes minimum, maximum, addition, multiplication, subtraction of real numbers respectively.

Definition 2.6 (Rough Set) [3] Let R be an equivalence relation on the universal set U. Then, the pair (U, R) is called a Pawlak's approximation space. An equivalence class of R containing x will be denotes by $[x]_R$. Now, for $X \subseteq U$, the upper and lower approximation of X with the respect to (U, R) are denoted by, respectively $A_1(x)$ and $A_2(x)$ and defined by

$$A_1(x) = \{x : [x]_R \subseteq X\} \text{ and } A_2(x) = \{x : [x]_R \cap X \neq \emptyset\}$$

Now, if $A_1(x) = A_2(x)$, then X is called definable; otherwise, the pair $A(X) = (A_1(x), A_2(x))$ is called the rough set of X in U.

Example 2.7 [5] Let $A =(U, R)$ be an approximate space where $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and the relation R on U be definable aRb iff $a \equiv b \pmod 5$ for all $a, b \in U$. Let us consider a subset $X = \{1, 2, 6, 7, 8, 9\}$ of U. Then, the rough set of X is $A(x) = (\underline{A}(x), \overline{A}(x))$ where $\underline{A}(x) = \{1, 2, 6, 7\}$ and $\overline{A}(x) = \{1, 2, 3, 4, 6, 7, 8, 9\}$. Here, the equivalence classes are

$[0]_R = [5]_R = [10]_R = \{0, 5, 10\}$

$[1]_R = [6]_R = \{1, 6\}$

$[2]_R = [7]_R = \{2, 7\}$

$[3]_R = [8]_R = \{3, 8\}$

$[4]_R = [9]_R = \{4, 9\}$

Thus, $\underline{A}(x) = \{x \in U: [x]_R \subseteq X\} =\{1, 2, 6, 7\}$ and

$\overline{A}(x) = \{x : [x]_R \cap X \neq \emptyset\}= \{1, 2, 3, 4, 6, 7, 8, 9\}$.

The following definitions are refer to [15].

Definition 2.8 Let $A = (A_1,A_2)$ and $B = (B_1,B_2)$ be two rough sets in the approximation space $S = (U, R)$. Then,

(i) $A \cup B = (A_1 \cup B_1, A_2 \cup B_2)$,

(ii) $A \cap B = (A_1 \cap B_1, A_2 \cap B_2)$,

(iii) $A \subseteq B$ if $A \cap B = A$,

(iv) $\sim A = \{U - A_2, U - A_1\}$.

Definition 2.9 (Rough Neutrosophic Set) Let U be a non-null set and R be an equivalence relation on U. Let A be neutrosophic set in U with the membership function T_A, indeterminacy function I_A and non-membership function F_A. The lower and the upper approximations of A in the approximation (U, R) denoted by $\underline{N}(A)$ and $\overline{N}(A)$ are respectively defined as follows:

$$\underline{N}(A) = \{\langle x, (T_A(x), I_A(x), F_A(x))\rangle \mid y \in [x]_R, x \in U\},$$

$$\overline{N}(A) = \{\langle x, (T_A(x), I_A(x), F_A(x))\rangle \mid y \in [x]_R, x \in U\}$$

where

$$T_{\underline{N}(A)}(x) = \bigwedge_{y\in[x]_R} T_A(y),\ \ I_{\underline{N}(A)}(x) = \bigvee_{y\in[x]_R} I_A(y),$$
$$F_{\underline{N}(A)}(x) = \bigvee_{y\in[x]_R} F_A(y)$$

$$T_{\overline{N}(A)}(x) = \bigvee_{y\in[x]_R} T_A(y),\ \ I_{\overline{N}(A)}(x) = \bigwedge_{y\in[x]_R} I_A(y),$$
$$F_{\overline{N}(A)}(x) = \bigwedge_{y\in[x]_R} F_A(y)$$

So,

$$0 \le T_{\underline{N}(A)}(x) + I_{\underline{N}(A)}(x) + F_{\underline{N}(A)}(x) \le 3, \quad \text{and}$$

$$0 \le T_{\overline{N}(A)}(x) + I_{\overline{N}(A)}(x) + F_{\overline{N}(A)}(x) \le 3$$

Here $\wedge$ and $\vee$ denote "min" and "max'' operators respectively. $T_A(y)$, $I_A(y)$ and $F_A(y)$ are the membership, indeterminacy and non-membership degrees of y with respect to A. $\underline{N}(A)$ and $\overline{N}(A)$ are two neutrosophic sets in U.

Thus, NS mappings $\underline{N}, \overline{N} : N(U) \to N(U)$ are, respectively, referred to as the upper and lower rough NS approximation operators, and the pair is $(\underline{N}(A), \overline{N}(A))$ called the rough neutrosophic set in (U, R).

Based on the above definition, it is observed that $\underline{N}(A)$ and $\overline{N}(A)$ have a constant membership on the equivalence classes of U, if $\underline{N}(A) = \overline{N}(A)$; i.e.,

$$T_{\underline{N}(A)}(x) = T_{\overline{N}(A)}(x),\ \ I_{\underline{N}(A)}(x) = I_{\overline{N}(A)}(x),$$
$$F_{\underline{N}(A)}(x) = I_{\overline{N}(A)}(x)$$

For any $x \in U$, A is called a definable neutrosophic set in the approximation (U, R). Obviously, zero neutrosophic set (0_N) and unit neutrosophic sets (1_N) are definable neutrosophic sets. Let consider the example in the following.

Example 2.10 Let $U = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8\}$ be the universe of discourse. Let R be an equivalence relation its partition of U is given by

$$U/R = \{\{p_1, p_4\}, \{p_2, p_3, p_6\}, \{p_5\}, \{p_7, p_8\}\}.$$

Let $N(A) = \{(p_1, (0.3, 0.4, 0.5), (p_4, (0.4, 0.6, 0.5)), (p_5, (0.5, 0.7, 0.3)), (p_7, (0.2, 0.4, 0.6))\}$ be a neutrosophic set of U. By definition 2.6 and 2.9, we obtain:

$\underline{N}(A) = \{(p_1, (0.3, 0.6, 0.5)), (p_4, (0.3, 0.6, 0.5)), (p_5, (0.5, 0.7, 0.3))\}$ and

$\overline{N}(A) = \{(p_1, (0.3, 0.4, 0.5)), (p_4, (0.3, 0.4, 0.5)), (p_5, (0.5, 0.7, 0.3)), (p_7, (0.2, 0.4, 0.6)), (p_8, (0.2, 0.4, 0.6))\}$.

For another neutrosophic sets,

$N(B) = \{(p_1, (0.3, 0.4, 0.5), (p_4, (0.3, 0.4, 0.5)), (p_5, (0.5, 0.7, 0.3))\}$.

The lower approximation and upper approximation of $N(B)$ are calculated as

$\underline{N}(B) = \{(p_1, (0.3, 0.4, 0.5)), (p_4, (0.3, 0.4, 0.5)), (p_5, (0.5, 0.7, 0.3))\}$ and

$\overline{N}(B) = \{(p_1, (0.3, 0.4, 0.5)), (p_4, (0.3, 0.4, 0.5)), (p_5, (0.5, 0.7, 0.3))\}$.

Obviously, $\underline{N}(B) = \overline{N}(B)$ is a definable neutrosophic set in the approximation space (U, R).

Definition 2.11 If $N(A) = (\underline{N}(A), \overline{N}(A))$ is a rough neutrosophic set in (U, R), the rough complement of N(A) is the rough neutrosophic set denoted by $\sim N(A) = (\underline{N}(A)^c, \overline{N}(A)^c)$ where $\underline{N}(A)^c$, $\overline{N}(A)^c$ are the complements of neutrosophic sets $\underline{N}(A)$ and $\overline{N}(A)$ respectively,

$$\underline{N}(A)^c = \{\langle x, F_{\underline{N}(A)}(x), 1 - I_{\underline{N}(A)}(x), T_{\underline{N}(A)}(x)\rangle \mid x \in U\},$$

$$\overline{N}(A)^c = \{\langle x, F_{\overline{N}(A)}(x), 1 - I_{\overline{N}(A)}(x), T_{\overline{N}(A)}(x)\rangle \mid x \in U\}$$

Definition 2.12 If $N(F_1)$ and $N(F_2)$ are two rough neutrosophic set of the neutrosophic sets F_1 and F_2 respectively in U, then we define the following:

(i) $N(F_1) = N(F_2)$ iff $\underline{N}(F_1) = \underline{N}(F_2)$ and $\overline{N}(F_1) = \overline{N}(F_2)$

(ii) $N(F_1) \subseteq N(F_2)$ iff $\underline{N}(F_1) \subseteq \underline{N}(F_2)$ and $\overline{N}(F_1) \subseteq \overline{N}(F_2)$

(iii) $N(F_1)\bigcup N(F_2) = \left\langle \underline{N}(F_1)\bigcup \underline{N}(F_2), \overline{N}(F_1)\bigcup \overline{N}(F_2)\right\rangle$

(iv) $N(F_1)\bigcap N(F_2) = \left\langle \underline{N}(F_1)\bigcap \underline{N}(F_2), \overline{N}(F_1)\bigcap \overline{N}(F_2)\right\rangle$

(v) $N(F_1)+N(F_2) = \left\langle \underline{N}(F_1)+\underline{N}(F_2), \overline{N}(F_1)+\overline{N}(F_2)\right\rangle$

(vi) $N(F_1)\cdot N(F_2) = \left\langle \underline{N}(F_1)\cdot \underline{N}(F_2), \overline{N}(F_1)\cdot \overline{N}(F_2)\right\rangle$

If N, M, L are rough neutrosophic set in (U, R), then the results in the following proposition are straightforward from definitions.

Proposition 2.13.

(i) $\sim N(\sim N) = N$

(ii) $N\bigcup M = M\bigcup N$, $N\bigcap M = M\bigcap N$

(iii) $(N\bigcup M)\bigcup L = N\bigcup(M\bigcup L)$, and $(N\bigcap M)\bigcap L = N\bigcap(M\bigcap L)$

(iv) $(N\bigcup M)\bigcap L = (N\bigcup M)\bigcap(N\bigcup L)$, and $(N\bigcap M)\bigcup L = (N\bigcap M)\bigcup(N\bigcap L)$

De Morgan's Laws are satisfied for rough neutrosophic sets:

Proposition 2.14.

(i) $\sim (N(F_1) \bigcup N(F_2)) = (\sim N(F_1)) \bigcap (\sim N(F_2))$

(ii) $\sim (N(F_1) \bigcap N(F_2)) = (\sim N(F_1)) \bigcup (\sim N(F_2))$

Proposition 2.15. If F_1 and F_2 are two neutrosophic sets in U such that $F_1 \subseteq F_2$, then $N(F_1) \subseteq N(F_2)$

(i) $N(F_1\bigcup F_2) \supseteq N(F_1)\bigcup N(F_2)$

(ii) $N(F_1\bigcap F_2) \subseteq N(F_1)\bigcap N(F_2)$

Proposition 2.16.

(i) $N(F) = \sim \overline{N}(\sim F)$

(ii) $\overline{N}(\sim F) = \sim \underline{N}(\sim F)$

(iii) $\underline{N}(F) \subseteq \overline{N}(F)$

3 Rough Neutrosophic Multisets

Based on the equivalence relation on the universe of discourse, we introduce the lower and upper approximations of neutrosophic multisets [20] in a Pawlak's approximation space [3] and obtained a new notion called rough neutrosophic multisets (RNM). Its basic operations such as complement, union and intersection also discuss over them with the examples. Some of it is quoted from [15], [25],[20], [26].

Definition 3.1 Let U be a non-null set and R be an equivalence relation on U. Let A be neutrosophic multisets in U with the truth membership sequence T_A^i, indeterminacy membership sequences I_A^i and falsity membership sequences F_A^i. The lower and the upper approximations of A in the approximation (U, R) denoted by $\underline{Nm}(A)$ and $\overline{Nm}(A)$ are respectively defined as follows:

$$\underline{Nm}(A) = \{\langle x, (T^i_{\underline{Nm}(A)}(x),\ I^i_{\underline{Nm}(A)}(x),\ F^i_{\underline{Nm}(A)}(x))\rangle \mid y \in [x]_R,\ x \in U\},$$

$$\overline{Nm}(A) = \{\langle x, (T^i_{\overline{Nm}(A)}(x),\ I^i_{\overline{Nm}(A)}(x),\ F^i_{\overline{Nm}(A)}(x))\rangle \mid y \in [x]_R,\ x \in U\},$$

where

$i = 1, 2, .., p,$

$$T^i_{\underline{Nm}(A)}(x) = \bigwedge_{y\in[x]_R} T^i_A(y),$$

$$I^i_{\underline{Nm}(A)}(x) = \bigvee_{y\in[x]_R} I^i_A(y),$$

$$F^i_{\underline{Nm}(A)}(x) = \bigvee_{y\in[x]_R} F^i_A(y),$$

$$T^i_{\overline{Nm}(A)}(x) = \bigvee_{y\in[x]_R} T^i_A(y),$$

$$I^i_{\overline{Nm}(A)}(x) = \bigwedge_{y\in[x]_R} I^i_A(y),$$

$$F^i_{\overline{Nm}(A)}(x) = \bigwedge_{y\in[x]_R} F^i_A(y)$$

such that,

$$T^i_{\underline{Nm}(A)}(x),\ I^i_{\underline{Nm}(A)}(x),\ F^i_{\underline{Nm}(A)}(x) \in [0,1],$$

$$T^i_{\overline{Nm}(A)}(x), I^i_{\overline{Nm}(A)}(x), F^i_{\overline{Nm}(A)}(x) \in [0,1],$$

$$0 \le T^i_{\underline{Nm}(A)}(x) + I^i_{\underline{Nm}(A)}(x) + F^i_{\underline{Nm}(A)}(x) \le 3, \text{ and}$$

$$0 \le T^i_{\overline{Nm}(A)}(x) + I^i_{\overline{Nm}(A)}(x) + F^i_{\overline{Nm}(A)}(x) \le 3$$

Here $\wedge$ and $\vee$ denote "min" and "max'' operators respectively. $T_A^i(y)$, $I_A^i(y)$ and $F_A^i(y)$ are the membership sequences, indeterminacy sequences and non-membership sequences of y with respect to A and i =1, 2, …, p.

Since $\underline{Nm}(A)$ and $\overline{Nm}(A)$ are two neutrosophic multisets in U, thus neutrosophic multisets mappings $\underline{Nm}, \overline{Nm} : Nm(U) \to Nm(U)$ are respectively referred to as the upper and lower rough neutrosophic multisets approximation operators, and the pair is $(\underline{Nm}(A), \overline{Nm}(A))$ called the rough neutrosophic multisets in (U, R).

From the above definition, we can see that $\underline{Nm}(A)$ and $\overline{Nm}(A)$ have constant membership on the equivalence classes of U, if $\underline{Nm}(A) = \overline{Nm}(A)$; i.e.,

$$T^i_{\underline{Nm}(A)}(x) = T^i_{\overline{Nm}(A)}(x),$$

$$I^i_{\underline{Nm}(A)}(x) = I^i_{\overline{Nm}(A)}(x),$$

$$F^i_{\underline{Nm}(A)}(x) = F^i_{\overline{Nm}(A)}(x).$$

Let consider the following example.

Example 3.2 Let $U = \{ p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8 \}$ be the universe of discourse. Let R be an equivalence relation its partition of U is given by

$U/R = \{\{p_1, p_4\}, \{p_2, p_3, p_6\}, \{ p_5\}, \{ p_7, p_8\}\}$.

Let $Nm(A)$ = {(<p_1, (0.8, 0.6, 0.5), (0.3, 0.2, 0.1), (0.4, 0.2, 0.1)>, <p_4, (0.5, 0.4, 0.3), (0.4, 0.4, 0.3), (0.6, 0.3, 0.3)>), <p_5, (0.2, 0.1, 0.0), (0.3, 0.2, 0.2), (0.8, 0.7, 0.7)>, <p_7, (0.7, 0.6, 0.5), (0.3, 0.2, 0.1), (0.4, 0.3, 0.2)> } be a neutrosophic multisets of U. By definition 3.1 we obtain:

$\underline{Nm}(A) = \{p_1, p_4, p_5\}$

={<p_1, (0.5, 0.6, 0.5), (0.3, 0.4, 0.3), (0.4, 0.3, 0.3)>, <p_4, (0.5, 0.6, 0.5), (0.3, 0.4, 0.3), (0.4, 0.3, 0.3)>, <p_5, (0.2, 0.1, 0.0), (0.3, 0.2, 0.2), (0.8, 0.7, 0.7)>} and

$\overline{Nm}(A) = \{p_1, p_4, p_5, p_7, p_8\}$

{<p_1, (0.8, 0.4, 0.3), (0.4, 0.2, 0.1), (0.6, 0.2, 0.1)>, <p_4, (0.8, 0.4, 0.3), (0.4, 0.2, 0.1), (0.6, 0.2, 0.1)>, <p_5, (0.2, 0.1, 0.0), (0.3, 0.2, 0.2), (0.8, 0.7, 0.7)>, <p_7, (0.7, 0.6, 0.5), (0.3, 0.2, 0.1), (0.4, 0.3, 0.2)>, <p_8, (0.7, 0.6, 0.5), (0.3, 0.2, 0.1), (0.4, 0.3, 0.2)>}.

For another neutrosophic multisets

$Nm(B)$ = {<p_1, (0.8, 0.6, 0.5), (0.3, 0.2, 0.1), (0.4, 0.2, 0.1)>, <p_4, (0.5, 0.4, 0.3), (0.4, 0.4, 0.3), (0.6, 0.3, 0.3)>, <p_5, (0.2, 0.1, 0.0), (0.3, 0.2, 0.2), (0.8, 0.7, 0.7)>}.

The lower approximation and upper approximation of $Nm(B)$ are calculated as

$\underline{Nm}(B) = \{p_1, p_4, p_5\}$

={<p_1, (0.5, 0.6, 0.5), (0.3, 0.4, 0.3), (0.4, 0.3, 0.3)>, <p_4, (0.5, 0.6, 0.5), (0.3, 0.4, 0.3), (0.4, 0.3, 0.3)>, <p_5, (0.2, 0.1, 0.0), (0.3, 0.2, 0.2), (0.8, 0.7, 0.7)>} and

$\overline{Nm}(B) = \{p_1, p_4, p_5\}$

={<p_1, (0.5, 0.6, 0.5), (0.3, 0.4, 0.3), (0.4, 0.3, 0.3)>, <p_4, (0.5, 0.6, 0.5), (0.3, 0.4, 0.3), (0.4, 0.3, 0.3)>, <p_5, (0.2, 0.1, 0.0), (0.3, 0.2, 0.2), (0.8, 0.7, 0.7)>}

Obviously, $\underline{Nm}(B) = \overline{Nm}(B)$ is a definable neutrosophic multisets in the approximation space (U, R).

Definition 3.3 Let $Nm(A) = (\underline{Nm}(A), \overline{Nm}(A))$ be a rough neutrosophic multisets in (U, R). The rough complement of $Nm(A)$ is denoted by $\sim Nm(A) = (\underline{Nm}(A)^c, \overline{Nm}(A)^c)$ where $\underline{Nm}(A)^c$ and $\overline{Nm}(A)^c$ are the complements of neutrosophic multisets of $\underline{Nm}(A)$ and $\overline{Nm}(A)$ respectively,

$$\underline{Nm}(A)^c = \{\langle x, (F^i_{\underline{Nm}(A)}(x) ,\ 1 - I^i_{\underline{Nm}(A)}(x) , T^i_{\underline{Nm}(A)}(x))\rangle \mid x \in U\},$$

$$\overline{Nm}(A)^c = \{\langle x, (F^i_{\overline{Nm}(A)}(x) ,\ 1 - I^i_{\overline{Nm}(A)}(x) , T^i_{\overline{Nm}(A)}(x))\rangle \mid x \in U\}$$

where i = 1, 2, …, p.

Example 3.4 Consider RNM, $Nm(A)$ in the set $X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$, $y \in [x]_R$ is equivalence relation and i = 1, 2, 3.

Let $Nm(A)$ = {⟨x_1, [(0.6, 0.4, 0.4), (0.7, 0.3, 0.4)], [(0.8, 0.4, 0.5), (0.7, 0.6, 0.5)], [(0.4, 0.3, 0.5), (0.3, 0.2, 0.7)] ⟩, ⟨ x_2, [(0.4, 0.3, 0.3), (0.5, 0.3, 0.4)], [(0.2, 0.4, 0.4), (0.3, 0.3, 0.5)], [(0.7, 0.8, 0.4), (0.7, 0.1, 0.5)]⟩, ⟨ x_4, [(0.2, 0.5, 0.7), (0.7, 0.8, 0.0)], [(1.0, 1.0, 0.0), (0.9, 0.2, 0.5)], [(0.1, 0.5, 0.3), (0.2, 0.8, 0.5)] ⟩}

Then the complement of $Nm(A)$ is defined as $\sim Nm(A) = (\underline{Nm}(A)^c, \overline{Nm}(A)^c) =$

{⟨x_1, [(0.4, 0.6, 0.6), (0.4, 0.7, 0.7)], [(0.5, 0.6, 0.8), (0.5, 0.4, 0.7)], [(0.5, 0.7, 0.4), (0.7, 0.8, 0.3)] ⟩, ⟨x_2, [(0.3, 0.7, 0.4), (0.4, 0.7, 0.5)], [(0.4, 0.6, 0.2), (0.5, 0.7, 0.3)], [(0.4, 0.2, 0.7), (0.5, 0.9, 0.7)]⟩, ⟨ x_4, [(0.7, 0.5, 0.2), (0.0, 0.2, 0.7)], [(0.0, 0.0, 1.0), (0.5, 0.8, 0.9)], [(0.3, 0.5, 0.1), (0.5, 0.2, 0.2)] ⟩}.

Definition 3.5 Let $Nm(A)$ and $Nm(B)$ are RNM respectively in U, then the following definitions hold:

(i) $Nm(A) = Nm(B)$ iff $\underline{Nm}(A) = \underline{Nm}(B)$ and $\overline{Nm}(A) = \overline{Nm}(B)$

(ii) $Nm(A) \subseteq Nm(B)$ iff $\underline{Nm}(A) \subseteq \underline{Nm}(B)$ and $\overline{Nm}(A) \subseteq \overline{Nm}(B)$

(iii) $Nm(A) \cup Nm(B) =$

$$\left\langle \underline{Nm}(A) \cup \underline{Nm}(B), \overline{Nm}(A) \cup \overline{Nm}(B) \right\rangle$$

(iv) $Nm(A) \cap Nm(B) =$

$$\left\langle \underline{Nm}(A) \cap \underline{Nm}(B), \overline{Nm}(A) \cap \overline{Nm}(B) \right\rangle$$

(v) $Nm(A) + Nm(B) =$

$$\left\langle \underline{Nm}(A) + \underline{Nm}(B), \overline{Nm}(A) + \overline{Nm}(B) \right\rangle$$

(vi) $Nm(A) \cdot Nm(B) =$

$$\left\langle \underline{Nm}(A) \cdot \underline{Nm}(B), \overline{Nm}(A) \cdot \overline{Nm}(B) \right\rangle$$

Example 3.6 Consider $Nm(A)$ in Example 3.4 and $Nm(B)$ are two RNM.

$Nm(B)$ = {⟨x_1, [(0.6, 0.1, 0.2), (0.3, 0.3, 0.3)], [(0.7, 0.2, 0.5), (0.8, 0.6, 0.5)], [(0.7, 0.3, 0.5), (1.0, 0.2, 0.7)] ⟩, ⟨ x_2, [(0.4, 0.4, 0.7), (0.6, 0.5, 0.6)], [(0.3, 0.4, 0.4), (0.6, 0.2, 0.5)], [(0.7, 0.8, 0.4), (0.6, 0.1, 0.5)]⟩, ⟨ x_3, [(0.3, 0.4, 0.5), (0.6, 0.4, 0.0)], [(1.0, 1.0, 0.0), (0.7, 0.2, 0.5)], [(0.1, 0.5, 0.3), (0.2, 0.8, 0.5)] ⟩, ⟨ x_4, [(0.4, 0.5, 0.6), (0.7, 0.8, 0.2)], [(1.0, 1.0, 0.0), (0.9, 0.2, 0.1)], [(0.6, 0.5, 0.3), (0.2, 0.2, 0.7)] ⟩}

Then, we have

(i) $Nm(A) \subseteq Nm(B)$

(ii) $Nm(A) \cup Nm(B)$

= {⟨x_1, [(0.6, 0.1, 0.2), (0.7, 0.3, 0.3)], [(0.8, 0.2, 0.5), (0.7, 0.6, 0.5)], [(0.7, 0.3, 0.5), (1.0, 0.2, 0.7)] ⟩, ⟨ x_2, [(0.4, 0.3, 0.3), (0.6, 0.3, 0.4)], [(0.3, 0.4, 0.4), (0.6, 0.2, 0.5)], [(0.7, 0.8, 0.4), (0.7, 0.1, 0.5)]⟩, ⟨ x_3, [(0.3, 0.4, 0.5), (0.6, 0.4, 0.0)], [(1.0, 1.0, 0.0), (0.7, 0.2, 0.5)], [(0.1, 0.5, 0.3), (0.2, 0.8, 0.5)]⟩, ⟨x_4, [(0.4, 0.5, 0.6), (0.7, 0.8, 0.0)], [(1.0, 1.0, 0.0), (0.9, 0.2, 0.1)], [(0.6, 0.5, 0.3), (0.2, 0.2, 0.5)] ⟩}.

(iii) $Nm(A) \cap Nm(B)$

= {⟨x_1, [(0.6, 0.4, 0.4), (0.3, 0.3, 0.4)], [(0.7, 0.4, 0.5), (0.7, 0.6, 0.5)], [(0.4, 0.3, 0.5), (0.3, 0.2, 0.7)]⟩, ⟨ x_2, [(0.4, 0.4, 0.7), (0.5, 0.5, 0.6)], [(0.2, 0.4, 0.4), (0.3, 0.3, 0.5)], [(0.7, 0.8, 0.4), (0.6, 0.1, 0.5)]⟩, ⟨x_3, [(0.3, 0.4, 0.5), (0.6, 0.4, 0.0)], [(1.0, 1.0, 0.0), (0.7, 0.2, 0.5)], [(0.1, 0.5, 0.3), (0.2, 0.8, 0.5)]⟩, ⟨x_4, [(0.2, 0.5, 0.7), (0.7, 0.8, 0.2)], [(1.0, 1.0, 0.0), (0.9, 0.2, 0.5)], [(0.1, 0.5, 0.3), (0.2, 0.8, 0.7)] ⟩}.

Proposition 3.7 If Nm, Mm, Lm are the RNM in (U, R), then the following propositions are stated from definitions.

(i) $\sim(\sim Nm) = Nm$

(ii) $Nm \cup Mm = Mm \cup Nm$, $Nm \cap Mm = Mm \cap Nm$

(iii) $(Nm \cup Mm) \cup Lm = Nm \cup (Mm \cup Lm)$, and $(Nm \cap Mm) \cap Lm = Nm \cap (Mm \cap Lm)$

(iv) $(Nm \cup Mm) \cap Lm = (Nm \cup Mm) \cap (Nm \cup Lm)$, and

$(Nm \cap Mm) \cup Lm = (Nm \cap Mm) \cup (Nm \cap Lm)$

Proof (i):

$\sim(\sim Nm(A)) = \sim(\sim(\underline{Nm}(A), \overline{Nm}(A)))$

$= \sim(\underline{Nm}(A)^c, \overline{Nm}(A)^c)$

$= (\underline{Nm}(A), \overline{Nm}(A))$

$= Nm(A)$

Proof (ii – iv) : The proofs is straightforward from definition.

Proposition 3.8 De Morgan's Law are satisfied for rough neutrosophic multisets:

(i) $\sim (Nm(A) \cup Nm(B)) = (\sim Nm(A)) \cap (\sim Nm(B))$

(ii) $\sim (Nm(A) \cap Nm(B)) = (\sim Nm(A)) \cup (\sim Nm(B))$

Proof (i):

$(Nm(A) \cup Nm(B))$

$= \sim (\{ \underline{Nm}(A) \cup \underline{Nm}(B) \}, \{ \overline{Nm}(A) \cup \overline{Nm}(B) \})$

$= (\sim\{ \underline{Nm}(A) \cup \underline{Nm}(B) \}, \sim\{ \overline{Nm}(A) \cup \overline{Nm}(B) \})$

$= (\{ \underline{Nm}(A) \cup \underline{Nm}(B) \}^c, \{ \overline{Nm}(A) \cup \overline{Nm}(B) \}^c)$

$= (\sim\{ \underline{Nm}(A) \cap \underline{Nm}(B) \}, \sim\{ \overline{Nm}(A) \cap \overline{Nm}(B) \})$

$= (\sim Nm(A)) \cap (\sim Nm(B))$.

Proof (ii): Similar to the proof of (i).

Proposition 3.9. If A and B are two neutrosophic multisets in U such that $A \subseteq B$, then $Nm(A) \subseteq Nm(B)$

(i) $Nm(A \cup B) \supseteq Nm(A) \cup Nm(B)$

(ii) $Nm(A \cap B) \subseteq Nm(A) \cap Nm(B)$

Proof (i):

$$T^i_{\underline{Nm}(A\cup B)}(x) = \inf\{T^i_{Nm(A\cup B)}(x) \mid x \in X\}$$

$$= \inf(\max\{T^i_{Nm(A)}(x), T^i_{Nm(B)}(x) \mid x \in X\})$$

$$= \max\{\inf\{T^i_{Nm(A)}(x) \mid x \in X\}, \inf\{T^i_{Nm(B)}(x) \mid x \in X\}\}$$

$$= \max\{(T^i_{\underline{Nm}(A)}(x), T^i_{\underline{Nm}(B)}(x)) \mid x \in X\}$$

$$= (T^i_{\underline{Nm}(A)} \cup T^i_{\underline{Nm}(B)})(x)$$

Similarly,

$$I^i_{\underline{Nm}(A\cup B)}(x) \le (I^i_{\underline{Nm}(A)} \cup I^i_{\underline{Nm}(B)})(x),$$

$$F^i_{\underline{Nm}(A\cup B)}(x) \le (F^i_{\underline{Nm}(A)} \cup F^i_{\underline{Nm}(B)})(x)$$

Thus, $\underline{Nm}(A \cup B) \supseteq \underline{Nm}(A) \cup \underline{Nm}(B)$

Hence,

$$Nm(A \cup B) \supseteq Nm(A) \cup Nm(B)$$

Proof (ii): Similar to the proof of (i).

Proposition 3.10.

(i) $\underline{Nm}(A) = \sim \overline{Nm}(\sim A)$

(ii) $\overline{Nm}(A) = \sim \underline{Nm}(\sim A)$

(iii) $\underline{Nm}(A) \subseteq \overline{Nm}(A)$

Proof (i): According to Definition 3.1, we can obtain

$$A = \{\langle x, (T^i_A(x), I^i_A(x), F^i_A(x))\rangle \mid x \in X\}$$

$$\sim A = \{\langle x, (F^i_A(x), 1 - I^i_A(x), T^i_A(x))\rangle \mid x \in X\}$$

$$\overline{Nm}(\sim A) = \{\langle x, (F^i_{\overline{Nm}(\sim A)}(x), 1 - I^i_{\overline{Nm}(\sim A)}(x), T^i_{\overline{Nm}(\sim A)}(x))\rangle \mid y \in [x]_R, x \in U\}$$

$$\sim \overline{Nm}(\sim A) = \{\langle x, (T^i_{\overline{Nm}(\sim A)}(x), 1 - (1 - I^i_{\overline{Nm}(\sim A)}(x)), F^i_{\overline{Nm}(\sim A)}(x))\rangle \mid y \in [x]_R, x \in U\}$$

$$= \{\langle x, (T^i_{\overline{Nm}(\sim A)}(x), I^i_{\overline{Nm}(\sim A)}(x), F^i_{\overline{Nm}(\sim A)}(x))\rangle \mid y \in [x]_R, x \in U\}$$

where

$$T^i_{\overline{Nm}(\sim A)}(x) = \bigwedge_{y\in[x]_R} T^i_A(y),$$

$$I^i_{\overline{Nm}(\sim A)}(x) = \bigvee_{y\in[x]_R} I^i_A(y),$$

$$F^i_{\overline{Nm}(\sim A)}(x) = \bigvee_{y\in[x]_R} F^i_A(y),$$

Hence $\underline{Nm}(A) = \sim \overline{Nm}(\sim A)$.

Proof (ii): Similar to the proof of (i).

Proof (iii): For any $y \in \underline{Nm}(A)$, we can have

$$T^i_{\underline{Nm}(A)}(x) = \bigwedge_{y\in[x]_R} T^i_A(y) \le \bigvee_{y\in[x]_R} T^i_A(y),$$

$$I^i_{\underline{Nm}(A)}(x) = \bigvee_{y\in[x]_R} I^i_A(y) \ge \bigwedge_{y\in[x]_R} I^i_A(y), \text{ and}$$

$$F^i_{\underline{Nm}(A)}(x) = \bigvee_{y\in[x]_R} F^i_A(y) \ge \bigwedge_{y\in[x]_R} F^i_A(y)$$

Hence $\underline{Nm}(A) \subseteq \overline{Nm}(A)$.

Conclusion

This paper firstly defined the rough neutrosophic multisets (RNM) theory and their properties and operations were studied. The RNM are the extension of rough neutrosophic sets [15]. The future work will cover the others operation in rough set, neutrosophic multisets and rough neutrosophic set that is suitable for RNM theory such as the notion of inverse, symmetry, and relation.

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